

1.6

Focus on Reasoning and Proving

People need to use their reasoning skills to make mathematical observations (such as those from patterns), to justify conclusions, and to use mathematics to plan and support solutions to problems.



Investigate

How can you apply reasoning skills to solve a problem?

A vending machine has 90 coins in its coin-collecting slots. There are one third as many dimes as quarters and half as many dimes as nickels. How much money is in the machine?

Understand the Problem

Choose a Strategy

Carry Out the Strategy

Reflect

1. Read the problem above. Read it again. Express it in your own words.
2. A possible strategy is to guess the answer, check to see if it works, and then revise the guess if necessary. What type of number must the number of nickels be? What type of number must the number of quarters be?
3. Make your first guess. If it is incorrect, read the clues again and use reasoning skills to decide what needs changing. Continue until you are successful.
4. Check that your answer works for each clue. Is another answer possible?
5. **Reflect** Was this the appropriate strategy for this problem? Explain.

conjecture

- a general conclusion drawn from a number of individual facts
- it may or may not be true

You may be asked to prove a mathematical **conjecture**. This means that you need to provide mathematical evidence that a statement is true.

Example Prove a Conjecture

Conjecture: The sum of any two consecutive whole numbers is an odd number.

- Give three examples of the conjecture.
- Prove that the conjecture is true.

Solution

- $1 + 2 = 3$
 $2 + 3 = 5$
 $3 + 4 = 7$

In these examples, the sum of two consecutive whole numbers is an odd number.

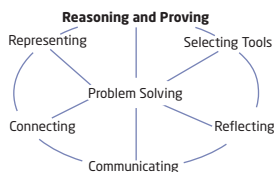
- Let n represent a whole number.
Then, $n + 1$ represents the next consecutive whole number.

The sum of these two consecutive numbers is

$$\begin{aligned} n + n + 1 \\ = 2n + 1 \end{aligned}$$

Since $2n$ means 2 times any whole number, this is always an even number. When you add 1 to an even number, you get the next odd number.

So, the conjecture is true.



Key Concepts

- You can use reasoning to determine which mathematical process is best in a given situation.
- Many simple problems can be solved using reasoning by systematic trial, especially if you have not yet learned other mathematical methods.
- To prove a mathematical conjecture, you need to justify your conclusion by using a well-organized mathematical argument.

Communicate Your Understanding

- C1** Jay's method for solving the problem "Which power of -2 is closest to -300 ?" is shown.

$$(-2)^5 = -32$$

$$(-2)^6 = 64$$

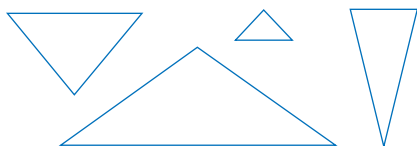
$$(-2)^7 = -128 \quad \Rightarrow \quad -300 - (-512) = 212$$

$$(-2)^8 = 256 \quad \Rightarrow \quad -300 - (-128) = -172$$

$$(-2)^9 = -512$$

The power of -2 that is closest to -300 is $(-2)^7$.

- a)** Describe Jay's strategy.
b) Do you think this was the most efficient strategy? Explain.
- C2 a)** Estimate the measure of each of the equal angles in the following triangles.

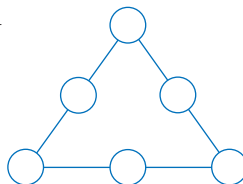


- b)** In each case, is your estimate greater than or less than 90° ?
c) Do your results prove that the equal angles of an isosceles triangle are acute? Explain.

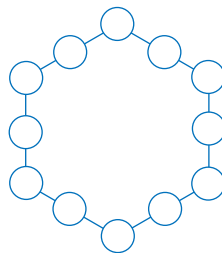
Practise

For help with questions 1 and 2, see the Investigate.

1. Copy the diagram. Place one of the numbers from 1 to 6 in each circle so that the sum of the three numbers on each side is the same.



2. Copy the diagram. Place one of the integers from -6 to 5 in each circle so that the sum of the three numbers on each side is the same.



For help with questions 3 and 4, see the Example.

3. Prove that the sum of three consecutive whole numbers is divisible by 3.
 4. Prove that a newspaper always has an even number of pages.

Connect and Apply

In each problem, write one or two sentences to describe your strategy. Then, carry out your strategy and justify your solution. Reflect and decide if your strategy was an appropriate one.

5. Paul claims that you only need seven coins to be able to make any amount of money up to 50¢. Show that Paul is correct.

6. Copy the numbers in the order shown. Replace each ■ with some of the symbols $+$, $-$, \times , \div , $()$, and $=$ to make true statements.

a) $5 \blacksquare 2 \blacksquare 8 \blacksquare 3 \blacksquare 15$

b) $25 \blacksquare 5 \blacksquare 11 \blacksquare 25 \blacksquare 9$

c) $\frac{1}{2} \blacksquare \frac{1}{3} \blacksquare \frac{11}{12} \blacksquare \frac{1}{12}$

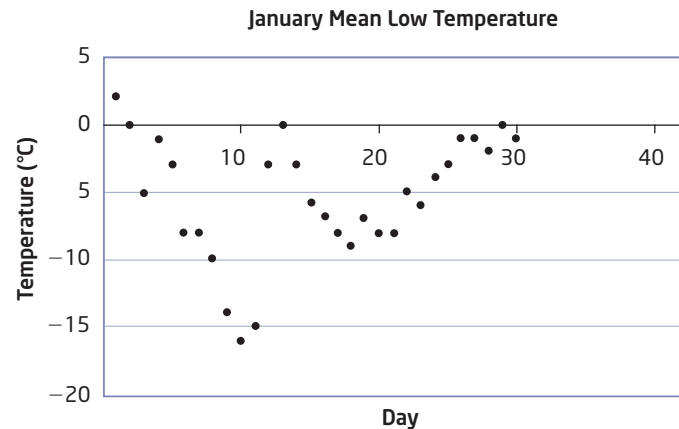
d) $\frac{2}{3} \blacksquare \frac{1}{8} \blacksquare \left(-\frac{1}{12}\right)$

mean

- the sum of the values in a set of data, divided by the number of values in the set



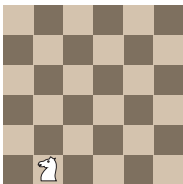
7. The **mean** low temperature between January 1 and January 30 was -5°C .



- a) What would the temperature need to be on January 31 to lower the mean temperature to -6°C ?
- b) What would the temperature need to be on January 31 to raise the mean temperature to -4.5°C ?

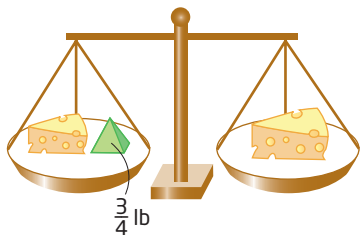
counter-example

- an example that proves that a conjecture is false



8. Give a **counter-example** to prove each statement false.
- All prime numbers are odd.
 - The sum of any two integers is always negative.
 - All fractions are less than 1.
 - All quadrilaterals are rectangles.
9. In the game of chess, a knight can move two spaces forward or backward and one space left or right, or two spaces left or right and one space forward or backward. Copy the small board shown. Can a knight eventually land on any square on the board?

10. Although most masses are measured in kilograms or grams in Canada, cheese is often bought by the pound. Three quarters of a pound and three quarters of a block of cheese balance with an entire block of cheese. What is the mass of the block of cheese, in pounds?



11. The integer -5 can be expressed as a difference of squares.

$$\begin{aligned} &2^2 - 3^2 \\ &= 4 - 9 \\ &= -5 \end{aligned}$$

How many integers between -1 and -10 can be expressed as a difference of squares of whole numbers?

12. Sam went on four rides a total of eight times at the fair. Roller Magic costs \$3.25, Death Drop costs \$3.75, The Amazing Loop costs \$4.00, and Fire Pit costs \$4.50. Sam says he went on one ride more than three times. He spent a total of \$33.00. How many times did Sam go on each ride?
13. How many floor tiles are there on the floors in your school?

Extend

14. Each three by three square, each row, and each column must contain each of the numerals 1 through 9 only once. Copy and complete this Sudoku puzzle.

		1	6	9		8		2
		7	5					
	9		2	4			7	
	2	5						
	1		8	5	9		2	
						9	5	
	7			1	5		9	
					3	2		
3		9		2	6	4		

15. What is the mass of a school bus with 45 student passengers?
16. For the sequence 1, -2 , -2 , 3, 3, 3, -4 , -4 , -4 , -4 , 5, 5, 5, 5, 5, ... ,
- what is the 50th term?
 - what is the 100th term?
 - what is the sum of the first 50 terms?
 - what is the sum of the first 100 terms?